Multi-Agent Based Adaptive Consensus Control for Multiple Manipulators with Kinematic Uncertainties

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Abstract—An adaptive control approach is proposed to deal with the multiple manipulators consensus problem based on the multi-agent theory. In the current multi-agent literature, agents were assumed to have determined models. However, the practical manipulator’s kinematics contains uncertain parameters. By using the projection method, the adaptive updating law for uncertain kinematic parameters is derived. Then, the estimated manipulator Jacobian matrix can be obtained to design the decentralized controller. By the proposed controller, all the manipulators’ end-effectors move toward the same configuration to achieve certain coordination tasks. In addition, the performance of the control system is analyzed by the Lyapunov method, and the consensus error is proved to approach zero. Finally, the effectiveness of the proposed scheme is illustrated by simulations on a multiple PUMA 560 robots system.

I. INTRODUCTION

Cooperative control of multiple manipulators has become an attractive area of research owing to its important role in the assembly automation and flexible manufacturing system. A great deal of strategies for this problem have been reported, which can be classified into three categories, the master-slave control [1], the centralized control [2], and the decentralized control [3]. Compared with the other two approaches, the decentralized one gets rid of the heavy computation load, and therefore has a larger application domain. It is also noted that there have been a number of studies on the multi-agent system coordination [4], which provides a general methodology for the interconnected multiple manipulators cooperative control.

On many occasions, all the end-effectors of manipulators need to reach a common configuration to fulfill certain tasks. In the multi-agent literature, it is called the consensus problem. There has been considerable effort in solving the general multi-agent consensus [5]–[12]. In [5], Jadbabaie et al. first investigated the angle synchronization of networks of integrator agents with switching topology. In [6], Olfati-Saber and Murray considered the average consensus of networks of first-order integrator agents with the directed information communication, and investigated the impact of network delay. In [7], Moreau studied a system of agents with nonlinear dynamics and switching communication links. The consensus problem was analyzed in the framework of set-valued Lyapunov theory. In [8], Hong et al. proposed a consensus protocol for multi-agent system with an active leader. It is demonstrated that every agent can track the leader’s trajectory even if partial velocity information of the leader agent is unknown. In [9], the multi-agent system with random measurement noises was taken into account. To achieve consensus, the quality of communication among agents needs to meet conditions determined by a convex optimization problem. In [10], Cao et al. suggested an asynchronous consensus protocol for multi-agent systems. Some graph-theoretic results were derived for the convergence analysis.

For the state of art of multi-agent consensus research, the readers are referred to [11], [12].

It is of interest to note that agents in most of existing work are assumed to have the exact model. To the best of the authors’ knowledge, there is no literature on the uncertainty of agent’s model. However, in the practical multiple manipulator coordination, the exact kinematic model of manipulator is hard to obtain due to the imprecision measurement of manipulator parameters and interactions between manipulator and different environments. As reported in [13], the research on the manipulator with uncertain kinematics is just a beginning. Recently, only two kinds of schemes were proposed to deal with the trajectory tracking of single manipulator with kinematic uncertainty: static approximation Jacobian control [14] and adaptive Jacobian method [15], [16].

This paper addresses the consensus problem of multiple manipulators with kinematic uncertainties. By the multi-agent theory and adaptive control method, a decentralized adaptive approach is proposed to control all the manipulators’ end-effectors to move towards the same configuration. Projection method is used to derive the adaptive updating law for uncertain kinematic parameters. Then the estimated manipulator Jacobian matrix can be obtained to facilitate the controller design. By the Lyapunov method, the consensus error is proved to approach zero.

The remainder of this paper is organized as follows. Section II introduces the problem formulation and some preliminary results. Section III provides the controller design. Section IV gives the the controller performance analysis. Illustrative simulation is given in Section V. Section VI concludes this paper with final remarks and discusses the future work.

Notations: for a vector \( v \in \mathbb{R}^n \), \( \|v\| \) denotes the \( l_2 \)-norm (Euclidean norm); for a matrix \( M \in \mathbb{R}^{n \times m} \), \( \|M\| \) denotes any matrix norm which is compatible with the vector \( l_2 \)-norm; \( I_n \) represents the \( n \)-dimensional unity matrix; \( \langle , \rangle \)
stands for the $i$th element of a given vector.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, a multi-manipulator system composed of $n$ rigid $m$-link, serially connected, direct-drive revolute robot manipulators is considered. According to the multi-agent theory [11], [12], the information exchange among manipulators can be modeled by a graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted graph with the set of nodes $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$, the set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = \{a_{ij}\}$ with non-negative adjacency elements. The node $v_i$ denotes the $i$th manipulator. An edge in $\mathcal{G}$ is denoted by an unordered pair $e_{ij} = (i,j)$. $e_{ij} \in \mathcal{E}$ if and only if there is the information exchange between manipulators $i$ and $j$, and $e_{ij} \in \mathcal{E} \Leftrightarrow e_{ji} \in \mathcal{E}$. The adjacency element $a_{ij}$ represents the communication quality between the $i$th and $j$th manipulators, that is $e_{ij} \in \mathcal{E} \Leftrightarrow a_{ij} > 0$. It is assumed that the communication quality between any two manipulators is the same, then $a_{ij} = a_{ji}$ such that $\mathcal{A}$ is a symmetric matrix.

The following definitions and results in the graph theory are useful to deal with the consensus problem. For more details, readers are referred to [17].

The Laplacian matrix $L$ of graph $\mathcal{G}$ is defined by

$$L = D - A,$$

(1)

where $D = \text{diag}(d_1, d_2, \cdots, d_n)$ and $d_i = \sum_{j=1}^{n} a_{ij}$.

A sequence of edges $(i_1, i_2), (i_2, i_3), \cdots, (i_{k-1}, i_k)$ is called a path from node $v_{i_1}$ to node $v_{i_k}$. If, for any two nodes $v_i, v_j \in \mathcal{V}$, there is a path between them, then $\mathcal{G}$ is called a connected graph. It is assumed that $\mathcal{G}$ studied in this paper is a connected graph.

Below is a well-known theorem for connected graphs.

**Theorem 1** ([12], [17]): If $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is an undirected connected graph, then the graph Laplacian matrix $L$ is a symmetric positive semi-definite matrix with $n$ real eigenvalues in an ascending order as

$$0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_n \leq C,$$

where $C = 2 \times (\max_{1 \leq i \leq n} d_i)$, and $\lambda_3$ is called the algebraic connectivity which can be used to analyze the convergence speed of consensus. In addition, $L \mathbf{1} = \mathbf{0}$, where $\mathbf{1} = (1, 1, \cdots, 1)^T \in \mathbb{R}^n$ and $\mathbf{0} = (0, 0, \cdots, 0)^T \in \mathbb{R}^n$.

According to [18], the forward kinematics equation of manipulator $i$ can be expressed as follows

$$x_i = h_i(q_i),$$

where $x_i \in \mathbb{R}^l$ denotes the task-space configuration vector of the $i$th manipulator’s end-effector; $q_i \in \mathbb{R}^m$ stands for the joint angle vector of the $i$th manipulator. The task-space velocity $\dot{x}_i$ is related to the joint-space velocity $\dot{q}_i$ as

$$\dot{x}_i = J_i(q_i, \phi_i)\dot{q}_i,$$

(2)

where $\phi_i \in \mathbb{R}^p$ represents the kinematic parameters of manipulator $i$, such as link lengths and joint offsets; $J_i(q_i, \phi_i) \triangleq (\partial h_i / \partial q_i) \in \mathbb{R}^{l \times m}$ denotes the manipulator’s Jacobian matrix which has the following property.

**Property 1:** The product of the manipulator Jacobian matrix with the joint velocity vector can be linearly parameterized as

$$J_i(q_i, \phi_i)\dot{q}_i = Y_i(q_i, \dot{q}_i)\phi_i,$$

(3)

where $Y_i(q_i, \dot{q}_i) \in \mathbb{R}^{l \times p}$ can be computed directly by the measurable joint position and velocity vectors $q_i$ and $\dot{q}_i$.

Now, the consensus problem can be formulated into designing the control law $\dot{q}_i$ for the $i$th manipulator to drive all the manipulators’ effectors to move towards a common task-space configuration, $x_i = x_j$, $1 \leq i, j \leq n$. In addition, $\dot{q}_i$ can only utilize the information of its connected manipulators.

III. CONTROLLER DESIGN

Consider the following control algorithm for the $i$th manipulator

$$\dot{q}_i = -k_iJ_i^+ (q_i, \phi_i) e_i + \left( I_m - J_i^+ \left( q_i, \phi_i \right) J_i \left( q_i, \phi_i \right) \right) \lambda_i,$$

(4)

where $J_i^+ \left( q_i, \phi_i \right) = \left( J_i^T \left( q_i, \phi_i \right) \left( J_i \left( q_i, \phi_i \right) J_i^T \left( q_i, \phi_i \right) \right)^{-1} \right.$

is the generalized inverse matrix of the approximate Jacobian matrix $J_i \left( q_i, \phi_i \right)$. $\phi_i$ is the estimation of uncertain kinematic parameter $\phi_i$, $k_i > 0$ is a positive constant. $\lambda_i \in \mathbb{R}^m$ is an auxiliary term which can be used for some optimization purposes when the manipulator is redundant. $e_i$ is defined by

$$e_i = \sum_{j=1}^{n} l_{ij} x_j,$$

(5)

where $l_{ij}$ is the $i$th row and the $j$th column element in the graph Laplacian matrix $L$.

It is assumed that the manipulator is operating in a finite task space such that the approximate Jacobian matrix is of full rank. This assumption is commonly adopted to deal with manipulator kinematic uncertainty in the existing literature [15], [16].

By Property 1, substituting (4) into (2) obtains

$$\dot{x}_i = \left( J_i(q_i, \phi_i) - J_i(q_i, \phi_i) \right) \dot{q}_i + \left( J_i(q_i, \phi_i) - J_i(q_i, \phi_i) \right) \dot{q}_i - k_i J_i(q_i, \phi_i) J_i^+ \left( q_i, \phi_i \right) e_i$$

$$+ J_i(q_i, \phi_i) \left( I_m - J_i^+ \left( q_i, \phi_i \right) J_i \left( q_i, \phi_i \right) \right) \lambda_i$$

$$= -k_i e_i + Y_i(q_i, \dot{q}_i) (\phi_i - \phi_i).$$

(6)

By the projection algorithm, the adaptive updating law for the kinematic parameter $\phi_i$ is derived as follows,

$$\dot{(\phi_i)}_j = \begin{cases} 
\beta_j ( Y_j^T (q_j, \dot{q}_j) e_j)_j , & \text{if } (\phi_i)_j \leq (\phi_i)_j \leq (\phi_i)_j \\
0 , & \text{if } (\phi_i)_j = (\phi_i)_j \\
\text{or if } (\phi_i)_j = (\phi_i)_j \text{ and } (Y_j^T (q_j, \dot{q}_j) e_j)_j > 0, \\
\text{or if } (\phi_i)_j = (\phi_i)_j \text{ and } (Y_j^T (q_j, \dot{q}_j) e_j)_j \leq 0; \\
\end{cases}$$

(7)
where $\beta_j$ is a positive scalar; $\phi_i^-$ and $\phi_i^+$ are the lower and upper bound vectors for estimated kinematic parameter $\hat{\phi}_i$, respectively. They are chosen to contain the real kinematic parameter $\phi_i$ in the bounded region, $(\phi_i^-)_j \leq (\phi_i)_j \leq (\phi_i^+)_j$.

It is emphasized that the initial value for $\hat{\phi}_i$ should be selected as

$$\left(\hat{\phi}_i(0)\right)_j \leq \left(\hat{\phi}_i(0)\right)_j \leq \left(\hat{\phi}_i^+(0)\right)_j.$$  \hspace{1cm} (8)

Remark 1: By the definition of graph Laplacian matrix $L$, it follows that $l_{ij} \neq 0$ if and only if the $i$th manipulator has information exchange with manipulator $j$. Therefore, by observing the proposed control method defined by (4) and the parameter updating law defined by (7), it is easy to see that the proposed controller for manipulator $i$ only uses the information of its connected manipulators. Hence, the proposed algorithm belongs to the decentralized design fashion with limited communications.

IV. PERFORMANCE ANALYSIS

Theorem 2: Given the multi-manipulator system composed of manipulators with kinematics defined by (2). If the decentralized controller is designed by (4), the uncertain kinematic parameter updating law is provided by (7), and the initial value for the estimated parameter satisfies (8), then all the manipulators’ end-effectors will approach the same task-space configuration, i.e., $x_i = x_j$, $1 \leq i, j \leq n$.

Proof: First, according to the projection method used in adaptive control [19], if the initial value for $\hat{\phi}_i(0)$ satisfies (8), then it easy to prove that $(\hat{\phi}_i^-)_j \leq (\hat{\phi}_i(0))_j \leq (\hat{\phi}_i^+)_j$.

That is, the estimated kinematic parameter $\hat{\phi}_i$ is limited in the given bounded region.

Consider the following Lyapunov function

$$E = \frac{1}{2} \sum_{i=1}^{n} \left(\hat{\phi}_i^T \Gamma^{-1} \hat{\phi}_i\right),$$  \hspace{1cm} (9)

where $x = (x_1^T, x_2^T, \cdots, x_T)^T \in \mathbb{R}^{nT}$, $\hat{\phi}_i = \phi_i - \hat{\phi}_i$, $\Gamma = \text{diag}(\beta_1, \beta_2, \cdots, \beta_p) \in \mathbb{R}^{p \times p}$, and $\otimes$ stands for the Kronecker product.

By (6), the time derivative of $E$ is

$$\frac{dE}{dt} = x^T(L \otimes I_1)x + \sum_{i=1}^{n} \left(\hat{\phi}_i^T \Gamma^{-1} \hat{\phi}_i\right)$$

$$= \sum_{i=1}^{n} \left(\sum_{j=1}^{n} l_{ij} x_j \right) \hat{x}_i - \left(\hat{\phi}_i^T \Gamma^{-1} \hat{\phi}_i\right)$$

$$= \sum_{i=1}^{n} \left(e_i^T - k_i e_i + Y_i(q_i, \hat{q}_i)\hat{\phi}_i\right) - \left(\hat{\phi}_i^T \Gamma^{-1} \hat{\phi}_i\right)$$

$$= -e^T K e + \sum_{i=1}^{n} \left(\hat{\phi}_i^T \left(Y_i^T(q_i, \hat{q}_i) e_i - \Gamma^{-1} \hat{\phi}_i\right)\right),$$  \hspace{1cm} (10)

where $e = (e_1^T, e_2^T, \cdots, e_n^T)^T \in \mathbb{R}^{nT}$, and $K = \text{diag}(k_1, k_2, \cdots, k_n)$.

By (7), it follows that

Case 1: if $\left(\dot{\hat{\phi}}_i\right)_j = \beta_j \left(Y_i^T(q_i, \hat{q}_i)e_i\right)_j$, then

$$\left(\ddot{\hat{\phi}}_i\right)_j \left(\left(Y_i^T(q_i, \hat{q}_i)e_i\right)_j - \frac{1}{\beta_j} (\hat{\phi}_i)_j\right) = 0;$$

Case 2: if $\left(\ddot{\hat{\phi}}_i\right)_j = 0$, then

$$\left(\ddot{\hat{\phi}}_i\right)_j \leq 0,$$  

or

$$\left(\ddot{\hat{\phi}}_i\right)_j > 0.$$  

By previous analysis and the fact that $(\phi_i^-)_j \leq (\hat{\phi}_i)_j \leq (\phi_i^+)_j$. Therefore,

$$\left(\ddot{\hat{\phi}}_i\right)_j \left(\left(Y_i^T(q_i, \hat{q}_i)e_i\right)_j - \frac{1}{\beta_j} (\hat{\phi}_i)_j\right)$$

$$= \left(\left(\hat{\phi}_i\right)_j - (\phi_i^-)_j\right) \left(\left(Y_i^T(q_i, \hat{q}_i)e_i\right)_j - \Gamma^{-1} (\hat{\phi}_i)_j\right) \leq 0.$$  \hspace{1cm} (11)

Hence, in both cases, it can be obtained that

$$\dot{\hat{\phi}}_i^T \left(Y_i^T(q_i, \hat{q}_i)e_i - \Gamma^{-1} \hat{\phi}_i\right) \leq 0.$$  \hspace{1cm} (12)

According to Theorem 1, it is obvious that 0 is an $l$-multiplicity eigenvalue of $(L \otimes I_1)$. And the eigenvectors associated with the eigenvalue 0 have the following form

$$q_1 = (v_1^T, v_1^T, \cdots, v_1^T)^T \in \mathbb{R}^{nT},$$

$$\cdots,$$

$$q_l = (v_l^T, v_l^T, \cdots, v_l^T)^T \in \mathbb{R}^{nT},$$

where $v_i \in \mathbb{R}^l$ is a vector whose $i$th element is $\frac{1}{\sqrt{n}}$ and the others are 0. Let $q_{m+1}, q_{m+2}, \cdots, q_{m+l}$ be another eigenvectors associated with the positive eigenvalues of $(L \otimes I_m)$ such that $q_1, q_2, \cdots, q_m$ can be set of orthogonal bases of $\mathbb{R}^{nT}$. Let $Q = (q_1, q_2, \cdots, q_m) \in \mathbb{R}^{nT \times nT}$, then $Q^T Q = Q Q^T = I_{nT}$ and $Q^T = Q^{-1}$.

It can be obtained that

$$x^T \left(L \otimes I_1\right) x = x^T Q^T \Lambda Q x,$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, \lambda_n)$. Let

$$\sqrt{\Lambda} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_{n-1}}, \sqrt{\lambda_n}),$$

$$\sqrt{\Lambda} = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \cdots, \sqrt{\lambda_{n-1}}, \sqrt{\lambda_n}),$$

and

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n-1}, \lambda_n).$$

Then,

$$x^T \left(L \otimes I_1\right) x = x^T Q^T \sqrt{\Lambda} \sqrt{\Lambda}^T Q x,$$

$$= x^T Q^T \Lambda Q^T \left(\Lambda^T\right)^{-1} \left(\Lambda^T\right)^{-1} \sqrt{\Lambda} \Lambda \Lambda Q x,$$

$$= x^T \Lambda Q^T Q \Lambda \Lambda Q x,$$

$$= x^T Q^T \Lambda Q x,$$

$$= x^T \left(L \otimes I_1\right)^T D \left(L \otimes I_1\right) x,$$

$$= e^T D e,$$
the Denavit and Hartenberg parameters of the PUMA 560 manipulator, where link lengths \( \alpha_2 = 0.4318 \text{m}, \alpha_3 = 0.0203 \text{m} \) and joint offsets \( d_3 = 0.15005 \text{m}, d_4 = 0.4318 \text{m} \). In the literature, these kinematic parameters, \( \alpha_2, \alpha_3, d_3 \) and \( d_4 \), are usually hard to determine according to the research in [20]. The initial joint configuration of each PUMA 560 robot is provided in Table II.

The control objective is to force all the manipulators’ end-effectors to the same position in the operating space. Because the last three joints of PUMA 560 robot do not contribute to the position of manipulator end-effector, there is no need to consider them in the controller design procedure. The relationship between the velocity of manipulator end-effector and the joint velocity can be expressed by

\[
\dot{x}_i = \begin{bmatrix} \dot{x}_{ix} \\ \dot{x}_{iy} \\ \dot{x}_{iz} \end{bmatrix} = J_i(q_i, \phi_i)q_i = Y_i(q_i, \dot{q}_i) \phi_i
\]

\[
J_i(q_i, \phi_i)_{11} = -a_3 s_1 c_{23} + d_3 c_1 - a_2 s_1 c_2 + d_4 s_1 s_{23}; \\
J_i(q_i, \phi_i)_{12} = -a_3 s_2 c_{11} - a_2 s_1 c_1 - d_4 c_1 c_{23}; \\
J_i(q_i, \phi_i)_{13} = -a_3 s_{23} c_1 - d_4 c_1 c_{23}; \\
J_i(q_i, \phi_i)_{21} = a_3 c_1 c_{23} + d_3 s_1 + a_2 c_1 c_2 - d_4 s_1 c_{23}; \\
J_i(q_i, \phi_i)_{22} = -a_3 s_1 s_{23} - a_2 s_1 s_2 - d_4 s_1 c_{23}; \\
J_i(q_i, \phi_i)_{23} = -a_3 s_1 s_{23} - d_4 s_1 c_{23}; \\
J_i(q_i, \phi_i)_{31} = 0; \\
J_i(q_i, \phi_i)_{32} = a_3 c_3 + a_2 c_2 - d_4 s_3; \\
J_i(q_i, \phi_i)_{33} = a_3 c_3 - d_4 s_3; \\
Y_i(q_i, \dot{q}_i)_{11} = s_1 c_1 c_{23} (\dot{q}_i)_1; \\
Y_i(q_i, \dot{q}_i)_{12} = s_1 c_1 c_{23} (\dot{q}_i)_1 - s_2 c_1 c_{23} (\dot{q}_i)_2 - s_3 c_1 c_{23} (\dot{q}_i)_3; \\
Y_i(q_i, \dot{q}_i)_{13} = c_1 (\dot{q}_i)_1; \\
Y_i(q_i, \dot{q}_i)_{21} = c_1 c_2 c_{13} (\dot{q}_i)_1 - s_1 s_2 c_{13} (\dot{q}_i)_2; \\
Y_i(q_i, \dot{q}_i)_{22} = c_1 c_2 c_{13} (\dot{q}_i)_1 - s_1 s_2 c_{13} (\dot{q}_i)_2 - s_1 s_3 c_{13} (\dot{q}_i)_3; \\
Y_i(q_i, \dot{q}_i)_{23} = s_1 c_1 (\dot{q}_i)_1; \\
Y_i(q_i, \dot{q}_i)_{31} = s_1 c_1 c_{23} (\dot{q}_i)_1 - s_1 c_1 c_{23} (\dot{q}_i)_2 - s_1 c_1 c_{23} (\dot{q}_i)_3; \\
Y_i(q_i, \dot{q}_i)_{32} = c_1 c_2 (\dot{q}_i)_2; \\
Y_i(q_i, \dot{q}_i)_{33} = c_1 c_2 (\dot{q}_i)_2 + c_2 c_3 (\dot{q}_i)_3; \\
Y_i(q_i, \dot{q}_i)_{33} = 0; \\
Y_i(q_i, \dot{q}_i)_{34} = -s_3 c_1 c_{23} (\dot{q}_i)_2 - s_3 c_1 c_{23} (\dot{q}_i)_3;
\]

where \( \phi_i = (\alpha_2, \alpha_3, d_3, d_4)^T \), \( s_1 = \sin((q_i)_1) \), \( s_2 = \sin((q_i)_2) \), \( s_{23} = \sin((q_i)_2 + (q_i)_3) \), \( c_1 = \cos((q_i)_1) \), \( c_2 = \cos((q_i)_2) \), \( c_{23} = \cos((q_i)_2 + (q_i)_3) \).

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Link} & \theta_1 \text{ (rad)} & \theta_2 \text{ (rad)} & \theta_3 \text{ (rad)} & \theta_4 \text{ (rad)} \\
\hline
1 & \pi/2 & 0 & 0 & 0 \\
2 & \pi/2 & 0 & 0 & 0 \\
3 & \pi/2 & 0 & \pi/2 & 0 \\
4 & \pi/2 & 0 & 0 & \pi/2 \\
5 & \pi/2 & 0 & 0 & 0 \\
6 & \pi/2 & 0 & 0 & 0 \\
\hline
\end{array}
\]

Table I: The Denavit and Hartenberg parameters of PUMA 560.

V. SIMULATION EXAMPLE

In this section, simulation examples are given to demonstrate the effectiveness of the proposed method. Manipulators’ kinematics defined by (2) is simulated by Matlab “ode15s” method.

Consider the multi-manipulator system composed of four industrial PUMA 560 robots. The information exchange among manipulators is shown in Fig. 1 with the following Laplacian matrix

\[
L = \begin{bmatrix}
0.40 & -0.10 & 0.00 & -0.30 \\
-0.10 & 0.30 & -0.20 & 0.00 \\
0.00 & -0.20 & 0.90 & -0.70 \\
-0.30 & 0.00 & -0.70 & 1.00
\end{bmatrix}.
\]

It is obvious that this graph is a connected one.

The mechanical configuration and coordinate system of each PUMA 560 robot are specified in [20]. Table I gives
The initial positions of four PUMA 560 end-effectors can be calculated as $x_1(0) = (0.0234, -0.1888, 0.2132)^T$m, $x_2(0) = (0.0217, -0.1697, 0.2289)^T$m, $x_3(0) = (-0.0683, -0.2351, 0.2351)^T$m, $x_4(0) = (0.0070, -0.2052, 0.1987)^T$m, respectively. The parameters in controller defined by (4) are set as $k_i = 30$, $\lambda_i = 0$, $(i = 1, 2, 3, 4)$. The parameters in kinematic parameter updating law defined by (7) are chosen as $\Gamma = \text{diag}(15, 15, 15, 15)$. $\phi_i = (0.01, 0.01, 0.01, 0.01)^T$, $\phi_i^+ = (5, 5, 5, 5)^T$. Figure 2 shows the trajectories of four PUMA 560 robots’ end-effectors. It can be obtained that all the manipulator can reach the steady consensus position $\bar{x} = (-0.0078, -0.2036, 0.2154)^T$. The profile of the distance between each robot's end-effector's position and the consensus position, $d_i = \|x_i(t) - \bar{x}\|$, is given in Fig. 3. The control variable, $\dot{q}_i$, for each PUMA 560 robot is shown in Fig. 4. According to the simulation results, the proposed control approach has a satisfactory performance.

VI. CONCLUSION AND FUTURE WORK

An adaptive control approach is proposed for multi-manipulator system consensus problem based on the multi-agent theory. However, most research so far in multi-agent consensus control has assumed that agents take the determined model. Unfortunately, due to the imprecision measurement and interactions with different environments, this assumption does not hold for the real multi-manipulator coordination. In this paper, the kinematic uncertainty of manipulator is taken into account. By employing the adaptive Jacobian method, the updating law for uncertain kinematic parameters is derived. System stability is analyzed by the Lyapunov method, and the consensus problem can be solved satisfactorily. Finally, the effectiveness of the proposed method has been validated by the simulation on the multiple PUMA 560 robots system.

It is noted that only the kinematic uncertainty is considered in this paper. In real high-precision manufacturing, the manipulator’s dynamics has to be incorporated into the controller design. The uncertainty in dynamics is more complicated than the uncertain kinematics. Fortunately, according to [15], [16], the uncertain dynamic parameters also satisfy the “linearity-in-parameters” condition. Therefore, the proposed approach can be extended to dealing with the uncertain dynamics by employing the backstepping technique and the adaptive scheme.

Also, in more practical case, the information exchange among manipulators may not be mutual. In other words, the graph weighted adjacency matrix $A$ is asymmetric. In the future work, much effort is going to be made to analyze the application possibility of the proposed method in the asymmetric case.

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Fig. 4. The control variable, \( \dot{q}_i \), for each PUMA 560 robot: (a) \( \dot{q}_1 \) (b) \( \dot{q}_2 \) (c) \( \dot{q}_3 \) (d) \( \dot{q}_4 \)


